TEMPERATURE FIELD IN METALS AFTER IMPINGEMENT OF THERMAL FLUX PULSES

S. N. Kapel'yan and Z. M. Yudovin

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The temperature field in a metal after impingement of high-power energy flux pulses is calculated, assuming that the acquired energy dissipates deeper into the metal by conduction and from the surface by radiation and convection. A solution is obtained in the form of asymptotic series for long and short time periods.

A variety of technical applications requires that the temperature field in a metal after an impingement of thermal flux pulses be known. The energy stored in the hot layer dissipates here by traveling deeper into the metal, by evaporating it, and by convection and radiation from its surface.

The corresponding two-dimensional thermophysics problem is, in the case of a semiinfinite body,

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}, \quad 0 < x, \ t < +\infty, \tag{1}$$

$$-\lambda \frac{\partial T}{\partial x}\bigg|_{x=0} = \sigma T^4|_{x=0} + hT|_{x=0}, \quad 0 < x < +\infty, \tag{2}$$

$$T(x,\ 0) = f(x), \quad 0 < t < +\infty, \tag{3}$$

$$\frac{\partial T}{\partial x}\bigg|_{x=\infty} = 0, \quad 0 < t < +\infty. \tag{4}$$

With the aid of Green's function, we obtain the integral equation

$$T(x,\ t) = \frac{1}{2\sqrt{\pi a t}}\int_0^\infty f(\xi)\left\{\exp\left[-\frac{(x-\xi)^2}{4at}\right] + \exp\left[-\frac{(x+\xi)^2}{4at}\right]\right\}d\xi$$
$$-b_1\int_0^t \frac{T^4(\tau)|_{x=0}}{\sqrt{t-\tau}}\exp\left[-\frac{x^2}{4a(t-\tau)}\right]d\tau - b_2\int_0^t \frac{T(\tau)|_{x=0}}{\sqrt{t-\tau}}\exp\left[-\frac{x^2}{4a(t-\tau)}\right]d\tau, \tag{5}$$

where $b_1 = (\sigma\sqrt{a})/(\lambda\sqrt{\pi})$ and $b_2 = (h\sqrt{a})/(\lambda\sqrt{\pi})$.

By approaching the limit $x \to 0$, we arrive at a nonlinear singular equation

$$\theta(t) = q(t) - b_1\int_0^t \frac{\theta^4(\tau)}{\sqrt{t-\tau}}\,d\tau - b_2\int_0^t \frac{\theta(\tau)}{\sqrt{t-\tau}}\,d\tau, \tag{6}$$

where $\theta(t) = T(0,\ t)$ and where $q(t)$ is determined by the temperature distribution at the initial instant of time

$$q(t) = \frac{1}{\sqrt{\pi a t}}\int_0^\infty f(\xi)\exp\left(-\frac{\xi^2}{4at}\right)d\xi. \tag{7}$$

A method has been developed in [1-2] of obtaining asymptotic expansions for an equation like (6) by analyzing the principal parts of the poles of the Mellin-transformation equation.

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Applying the Mellin transformation

$$T(s)=\int_0^\infty \theta(\tau)\tau^{s-1}d\tau,\quad T_4(s)=\int_0^\infty \theta^4(\tau)\tau^{s-1}d\tau,\quad Q(s)=\int_0^\infty q(\tau)\tau^{s-1}d\tau,$$

to Eq. (6) yields

$$T(s)=Q(s)-b_1\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}-s\right)}{\Gamma(1-s)}\left[T_4\left(s+\frac{1}{2}\right)+\frac{b_2}{b_1}T\left(s+\frac{1}{2}\right)\right], \qquad 0<\operatorname{Re}s<\frac{1}{2}, \tag{8}$$

where Q(s) is expressed in terms of the function $F(s)=\int_0^\infty f(\xi)\xi^{s-1}d\xi$, the Mellin transformant of the initial distribution f(x)

$$Q(s)=\frac{1}{\sqrt{\pi a}}(4a)^{-s+\frac{1}{2}}\Gamma\left(\frac{1}{2}-s\right)F(2s). \tag{9}$$

We will seek the asymptotic solution to (6) corresponding to short and long time periods respectively, in the form of logarithmic-power series [2].

The singularities of the coefficients in (9) lie in the right-hand half-plane and, for this reason, the structure of the solution in the left-hand half-plane depends only on the analytic behavior of F(s). The physical meaning of this is that, at small values of the time variable, $\theta(\tau)$ depends only on the behavior of f(x) at small values of x. If f(x) at small values of x is represented by a power series, then $\theta(t)$ can also be sought in the form of a power series

$$\theta(t)\sim\sum_i c_i t^{\nu_i},\qquad t\to 0. \tag{10}$$

In the right-hand s-half-plane the poles of the coefficient function can be superposed on the poles of the Mellin transformant of the unknown function. Such an increase in the multiplicity of poles is interpreted in the procedure of [2] as the appearance of logarithmic factors in the coefficients of inverse-power series, and the asymptoticity should be sought in the form

$$\theta(t)\sim\sum_i\frac{P_{m_i}(\ln t)}{t^{\nu_i}},\qquad t\to\infty, \tag{11}$$

where $P_{m_i}(\ln t)$ is an m_i-th power polynomial.

Let f(x) be an exponential relation. Such a temperature distribution occurs as a result of high-power energy flux pulses (ablation of the surface [3], pulse discharge [4], radiant fluxes [5]), when the metal in the active zone evaporates at a high rate and the evaporation front penetrates deeper at some stable velocity v determined by the thermal flux density. The two-dimensional thermophysics problem for this case has been analyzed in [3-6], where it also has been shown that the thermal field at the end of a rectangular pulse is described by the following expression:

$$f(x)=T_0\exp(-\beta x), \tag{12}$$

where $1/\beta = a/v$ is the characteristic dimension of the hot surface and T_0 denotes the temperature at the evaporation front, the latter being determined from the law of energy conservation. Numerical estimates in [7] indicate that high-rate evaporation continues for an extremely short time ($<10^{-5}$ sec) after the end of a thermal flux pulse in the 10^9-10^{10} W/m^2 range and that, to the first approximation, the temperature distribution is given by the solution to problem (1)-(4) with f(x) defined according to (12). For q(t) from (7) and (12) or, based on the correspondence of the poles of Q(s), from (9) respectively we have, considering the asymptoticity of q(t)

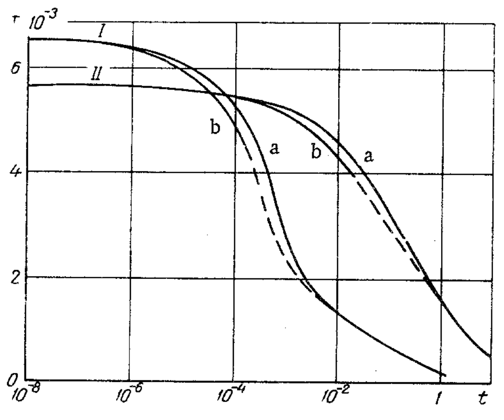

Fig. 1. Variation W of surface temperature after end of pulse: I) 10^{10} W/m^2; II) 10^9 W/m^2, without accounting for radiation (a), with radiation accounted for (b). Temperature T (°C), time t (sec).

$$q(t) \sim T_0 (a_0 + a_1 t^{1/2} + a_2 t + a_3 t^{3/2} + \ldots), \quad t \to 0, \tag{13}$$

$$q(t) \sim T_0 (d_1 t^{-1/2} + d_2 t^{-3/2} + d_3 t^{-5/2} + d_4 t^{-7/2} + \ldots), \quad t \to \infty, \tag{14}$$

where

$$a_0 = 1; \; a_1 = -\frac{2}{\sqrt{\pi}} (a\beta^2)^{1/2}; \; a_2 = a\beta^2; \; a_3 = -\frac{4}{3\sqrt{\pi}} (a\beta^2)^{3/2}, \; \ldots; \; d_1$$

$$= \frac{1}{\sqrt{\pi}\,(a\beta^2)^{1/2}}; \quad d_2 = -\frac{1}{2\sqrt{\pi}} \cdot \frac{1}{(a\beta^2)^{3/2}}, \quad d_3 = \frac{3}{4\sqrt{\pi}} \cdot \frac{1}{(a\beta^2)^{5/2}},$$

$$d_4 = -\frac{15}{8} \cdot \frac{1}{(a\beta^2)^{7/2}}, \ldots$$

In series (10) one easily discerns, according to (8) and (9), a half-powers sequence and, if series (10) is written as

$$\theta(t) \sim T_0 \sum_{i=0}^{\infty} (a_i - c_i)\, t^{i/2}, \quad t \to 0,$$

then the coefficients of the asymptotic series are

$$c_0 = 0, \quad c_1 = \frac{\sqrt{\pi}}{\Gamma\left(\frac{3}{2}\right)} (b_1 T_0^3 + b_2), \quad c_2 = \frac{\sqrt{\pi}\,\Gamma\left(\frac{3}{2}\right)}{\Gamma(2)} (4b_1 T_0^3 + b_2)(a_1 - c_1),$$

$$c_3 = \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{5}{2}\right)} \{b_1 T_0^3 [6(a_1 - c_1)^2 + 4(a_2 - c_2)] + b_2 (a_2 - c_2)\}$$

and the temperature field on the surface is

$$\frac{\theta(t)}{T_0} \sim 1 + (a_1 - c_1)\, t^{1/2} + (a_2 - c_2)\, t + (a_3 - c_3)\, t^{3/2} + \ldots \tag{15}$$

Inserting (12) and (15) into (5) yields for T(x, t)

$$T(x, t) \sim \frac{T_0}{2} \exp(a\beta^2 t) \left[\exp(-\beta x)\operatorname{erfc}\left(\beta\sqrt{at} - \frac{x}{2\sqrt{at}}\right)\right.$$

$$\left. + \exp(\beta x)\operatorname{erfc}\left(\beta\sqrt{at} + \frac{x}{2\sqrt{at}}\right)\right] - T_0 P_{\left(-\frac{3}{4}, \frac{1}{4}\right)}(x, t)(b_1 T_0^3 + b_2)$$

$$- T_0 P_{\left(-\frac{5}{4}, \frac{1}{4}\right)}(x, t)(a_1 - c_1)(4b_1 T_0^3 + b_2) - T_0 P_{\left(-\frac{7}{4}, \frac{1}{4}\right)}(x, t)\{b_1 T_0^3 [6(a_1 - c_1)^2 - 4(a_2 - c_2)] + b_2 (a_2 - c_2)\} + \ldots$$

$$P_{\left(-\frac{i}{4}, \frac{1}{4}\right)}(x, t) = \left(\frac{x^2}{4a}\right)^{-1/4} \exp\left(-\frac{x^2}{8at}\right) t^{i/4}\, \Gamma\left(\frac{i+1}{4}\right) W_{\left(-\frac{i}{4}, \frac{1}{4}\right)}\left(\frac{x^2}{4at}\right), \tag{16}$$

where $W_{(-i/4, 1/4)}(x^2/4at)$ is the Whittacker function (i = 3, 5, 7, ...).

Actual calculations have shown that the effect of convection is insignificant (of the order of 1%) even during short time periods.

As to the asymptoticity during long time periods, an analysis of the poles of Eq. (8) shows that $\theta(t)$ is

$$\theta(t) \sim \frac{1}{t^{1/2}} \sum_{n=0}^{\infty} \frac{P_n(\ln t)}{t^n}, \quad t \to \infty. \tag{17}$$

Only the first term of this expansion needs to be evaluated:

$$\theta(t) \sim \frac{A}{t^{1/2}}.$$

One determines it on the premise that, in accordance with the boundary condition (2) and the definition of the Mellin interval, $\{b_1T_4(1) + b_2T(1)\}$ represents the total flux passing through the metal surface during the time $(0, \infty)$, i.e., the total flux charge P received by the metal during heating. Therefore, at the $s = 1/2$ pole of Eq. (8)

$$A = \frac{T_0}{\sqrt{\pi}} (a\beta^2)^{-1/2} - AP, \quad A = \frac{T_0(a\beta^2)^{-1/2}}{\sqrt{\pi}(1+P)}.$$

The variation of surface temperature, with or without radiative heat losses taken into account, has been calculated for thermal fluxes $q = 10^9$-10^{10} W/m^2 and the results are shown in Fig. 1. The temperature of the metal surface at the end of a pulse is determined from the solution to the thermophysics problem in [8]. The most significant radiative losses in metals amount to 10-15% in terms of W.

Thus the basic mechanism of energy dissipation after impingement of thermal flux pulses on a metal appears to be evaporation in the first stage and then heat conduction. Radiative losses are comparable to conductive losses at higher thermal flux densities.

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